# Continuity and Change (Activity) Are Fundamentally Related In DEVS Simulation of Continuous Systems

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**Abstract.** The success of DEVS methods for simulating large continuous models calls for more in-depth examination of the applicability of discrete events in modeling continuous phenomena. We present a concept of event set and an associated measure of activity that fundamentally characterize discrete representation of continuous behavior. This metric captures the underlying intuition of continuity as well as providing a direct measure of the computational work needed to represent continuity on a digital computer. We discuss several application possibilities beyond high performance simulation such as data compression, digital filtering, and soft computation. Perhaps most fundamentally we suggest the possibility of dispensing with the mysteries of traditional calculus to revolutionize the prevailing educational paradigm.

## 1 Introduction

Significant success has been achieved with discrete event approaches to continuous system modeling and simulation[1,2,3]. Based on quantization of the state variables, such approaches treat threshold crossings as events and advance time on the basis of predicted crossings rather than at fixed time steps [4,5,6]. The success of these methods calls for more in-depth examination of the applicability of discrete events in modeling continuous phenomena. I have previously proposed that discrete events provide the right abstraction for modeling both physical and decision-making aspects of real-world systems. Recent research has defined the concept of activity which relates to the characterization and heterogeneous distribution of events in space and time. Activity is a measure of change in system behavior – when it is divided by a quantum gives the least number of events required to simulate the behavior with that quantum size. The number of DEVS model transitions, and hence the simulation execution time, are directly related to the threshold crossings. Hence activity is characteristic of continuous behaviors that lower bounds work needed to simulate it on a digital computer. The activity measure was originally formulated in the context of ordinary and partial differential equations as the integral of the magnitudes of the state space derivatives. This paper goes deeper into the activity measure to relate it to the information content of a system behavior and to the very concept of continuity itself.

The activity, re-examined, turns out to be a measure of variation defined on finite sets of events. The value of this measure will tend to increase as we add events. But what is critical is that, once we have enough events to get the qualitative characteristics of the curve, the measure slows down markedly or stops growing at all. Indeed, if we are lucky enough to start with the right set of events then the measure should stay constant from the very start of the refinement process. By qualitative characteristics of the curve we mean the placement of its minima and maxima and we restrict the curves of interest to those for which there are only a finite number of such extreme points in the finite interval of interest. We will show that for a continuous curve, for any initial sample set containing these points, the variation measure must remain constant as we continue to inject new samples. If the sample set does not include these extreme points, then the measure will grow rapidly until points are included that are close enough to these extrema. Since performing simulations with successively smaller quantum sizes generates successive refinements of this kind, we can employ this concept to judge when a quantum size is just small enough to give a true qualitative picture of the underlying continuous behavior. For spatially extended models, employing the measure for successively smaller cell sizes, gives the same characterization for resolution required for continuity in space.

## 2. Review of Activity Results

Models with large numbers of diverse components are likely to display significant heterogeneity in their components' rates of change. The activity concept that we developed is intended to exploit this heterogeneity by concentrating computing attention on regions of high rates of change – high activity – in contrast to uniformly attending to all component changes with indifference to their activity levels. The concept of activity, informally stated in this manner, applies to all heterogeneous milticomponent models, whether expressed in continuous or discrete formalisms. Our focus here however, is on elucidating the activity concept within the context of continuous systems described by differential equations with the goal of intimately linking the concept to discrete event simulation of such models. We have shown in recent work that activity can be given a very intuitive and straightforward definition in this context, that useful theoretical and practical implications can be derived, and that such implications can be verified with empirical computational results. Indeed, several studies have confirmed that using the quantization method of differential equation solution, DEVS simulation naturally, and automatically, performs the requisite allocation of attention in proportion to activity levels. It does so by assigning time advances inversely to rates of change, so that high rates of change get small time advances, while low rates of change get large time advances. Hence component events are scheduled for execution inversely to their activity levels. Furthermore, this occurs in a dynamic manner, tracking changes in rates in a natural way, “at no extra charge.”

### 2.1 Mapping ODEs to Quantized DEVS Networks

A mapping of ordinary differential equations (ODE) into DEVS integration networks using quantization is detailed in [5], where supporting properties such as completeness and error dependence on quantum size are established. Essentially, an ODE is viewed as a network of instantaneous functions and integrators that are mapped in a one-one manner into an equivalent coupled network of DEVS equivalents. Each integrator operates independently and asynchronously in that its time advance is computed as the quantum divided by the just-received input derivative. Such inputs are computed by instantaneous functions from outputs of integrators that they receive via coupling. Were such inputs to be the same for all integrators and to remain constant over time, then all integrators would be undergoing equal time advances, identical to the time steps of conventional numerical schemes as characterized by the Discrete Time System Specification (DTSS) formalism. However, such equality and constancy is not the norm. Indeed, as just mentioned above, we expect the DEVS simulation to exploit situations where there is considerable heterogeneity,

### 2.2 Activity Definition

We proceed to review the concept of activity as defined in [3], which should be consulted for more detailed discussion of the material in the next sub-sections. Fig 1 illustrates the concept of activity as a measure of the amount of computational work involved in quantized ODE simulations. Given equally spaced thresholds separated from each other by a quantum, the number of crossings that a continuous non-decreasing curve makes is given by the length of the range interval it has traveled divided by the size of the quantum. This number of threshold crossing is also the number of DEVS internal transitions that an integrator must compute. While the quantum size is an arbitrary choice of the simulationist, the range interval length is a property of the model, thus justifying the designation of this length to underlie our activity measure .Generalizing to a curve that has a finite number of alternating maximum and minima, we have the definition of activity in an interval

$$A = \sum_i | m_{i+1} - m_i | \qquad \textbf{(1)}$$

where $m_0, m_1, m_2..m_n$ the finite sequence of extrema of the curve in that interval. The number of threshold crossings, and hence the number of transitions of the quantized integrator, is then given by the activity divided by the selected quantum size

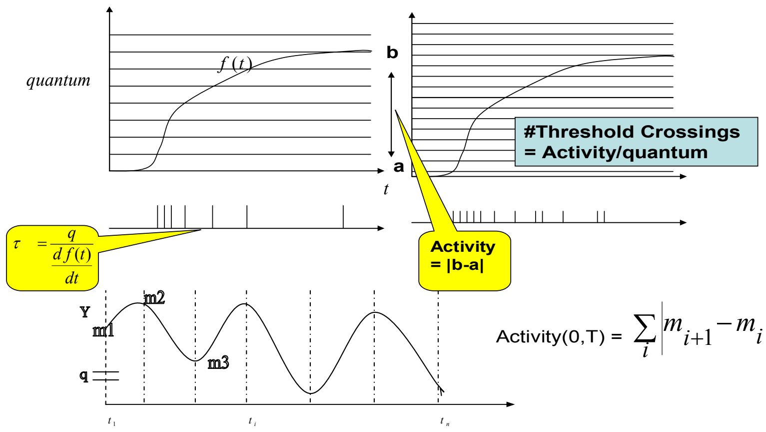



**Fig. 1.** Activity as a characteristic of continuous functions

### 2.3 Computing the Activity

The activity measure in Eqn. 1 relates back to quantized integration of an ordinary differential equation. The total number of DEVS transitions is the sum of those of the individual integrators. This number is predicted by the total activity divided by the quantum size, where total activity is the sum of individual activities of the integrators.

We can derive the rates of activity accumulation from the underlying ODE by noting that:

$$\frac{dy_i}{dt} = f_i(y_1 \ldots y_n)$$

$$\Rightarrow$$

$$\frac{dA_i}{dt} = | f_i(y_1 \ldots y_n) |$$

i.e., the instantaneous rate of activity accumulation at an integrator is the absolute value of its derivative input. When integrated over an interval, this instantaneous differential equation turns out to yield the accumulated activity expressed in (1).

The total activity of all integrators can be expressed as

$$A = \int \overline{f}(y_1 \ldots y_n)dt \qquad \textbf{(2)}$$

where

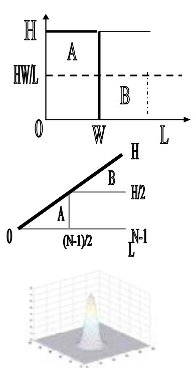
$$\overline{f}(y_1 \ldots y_n) = \sum_i | f_i(y_1 \ldots y_n) | .$$

We employ (2) to derive activity values where analytically possible in the next section.

### 2.4 Activity in Partial Differential Equation Models

The activity formula derived so far applies to ordinary differential equations. We extended it to partial differential equations (PDE) by discretizing space into a finite number of cells and approximating the PDE as an ODE whose dimension equals the number of cells. The activity formula (2) is than applied to this family of ODEs with parameter N, the number of cells. For a one-dimensional diffusion example, when the number of cells increases to infinity, the solution converges to that of the PDE and we found the activity likewise converges. Table 1 displays activity formulas for different initial states (diffusant distributions), where each formula gives the accumulated activity over all cells until equilibrium is reached.  In each case, the average activity (total divided by number of cells) approaches a constant whose value depends on the initial state.

**Table 1.** Activity calculation for one-dimensional diffusion with different initial conditions




| Initial state | Activity | Activity/N as N→∞ |
|---|---|---|
| Rectangular pulse | 2HN(W/L)(1 –W/L) | 2H(W/L) (1 –W/L) |
| Triangular pulse | (N-1)*H/4 | H/4 |
| Gaussian pulse | $\frac{N*H}{L}\left(\frac{2}{\sqrt{2*\pi*e}}\ln\left(\frac{L}{\sqrt{2*c*t_{start}}}\right) - \frac{1}{2}erf\left(\frac{L}{\sqrt{4*c*t_{start}}}\right) + erf(0.707)\right)$ | Constant/L |

### 2.5 Ratio of DTSS to DEVS Transitions

Fig. 2 illustrates how we compare the number of transitions required by a DTSS to that required by a quantized DEVS to solve the same PDE with the same accuracy. We derive the ratio:

$$\frac{\# DTSS}{\# DEVS} = \frac{MaxDeriv * T}{A / N} \geq 1 \qquad (3)$$

where $MaxDeriv = Max_i\left(\left|\frac{dy_i}{dt}\right|\right)$

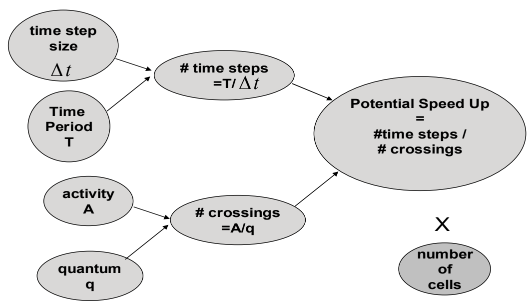



**Fig. 2.** Illustrating how to compare the number of DTTS transitions with those of DEVS for solutions with the same accuracy.

Table 2 shows the result of plugging in the activity calculations in Table 1 into the ratio formula for the initial states discussed earlier. We see that for the rectangular and triangular initial states, the ratio grows with the number of cells; while for the Gaussian pulse, the ratio grows with the length of the space in which the pulse is contained. The increasing advantage for increasing cell numbers was confirmed in work by Alexandre Muzy [7] who compared the execution times of the quantized DEVS with those of standard implicit and explicit methods for the same fire-spread model as shown in Fig 3

**Table 2.** Ratio of DTSS transitions to DEVS transitions for the same accuracy of solution

| **Initial state** | **#DTSS/#DEVS** |
|---|---|
| **Rectangular Pulse** | $\frac{TcN^2}{2w(1-w)L^2}$ where w is the width to the length ratio |
| **Triangular Pulse** | $\frac{4TcN}{L^2}$ |
| **Gaussian Pulse** | $\frac{0.062T}{(c * t_{start}{}^3)^{1/2}} f(L)$ where f is an increasing function of L |

We see that the growth of the quantized DEVS execution time with cell numbers is linear while that of the traditional methods is quadratic. Indeed, the fire-spread model employed in Muzy's results is characterized by a sharp derivative at ignition while otherwise obeying a diffusion law. This suggests that the predictions of the quadratic advantage for the quantized DEVS approach to the rectangular pulse in Table 2 apply.

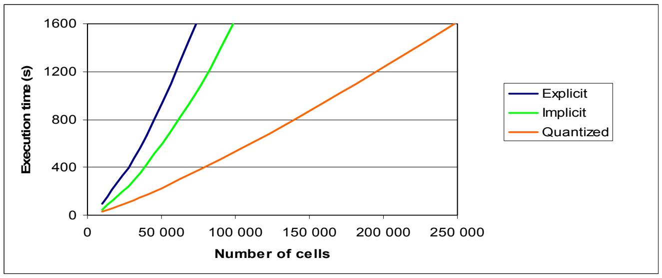



**Fig. 3.** Comparison of traditional DTSS methods with DEVS quantized integration (taken from [7] with permission)

This concludes our brief review of activity theory. For more details the reader is referred to the references at the end of the paper.

## 3 Basic Concepts of Event Sets

We now re-examine the activity concept from a more general point of view. We start with a discrete set theoretic formalism that has direct digital computer implementation. We introduce the concept of event set and event set refinement as an implementable approach to continuity. An *event set* is a finite ordered set of such event pairs,

$$E = \{(t_i, v_i) \mid i = 1...n\}$$

where the pairs are ordered in increasing order of the left hand elements of the pairs. Further the event set is a functional relation in that no two event pars have the same left hand value. In other words, we don't allow multiple events to be recorded at the same time or in the same place in the same event set – although we can use more than one event set to capture such simultaneous or co-located events. The following are needed

$domain(E) = \{t_i \mid i = 1...n\}$ and $range(E) = \{v_i \mid i = 1...n\}$

$size(E) = n$

The asymmetry between domain and range, succinctly encapsulated in the mathematical definition of function, will turn out to be an essential fact that we will later exploit to reduce simulation work. We are interested in the intervals that contain the respective domain and range points. Thus we define: $domainInterval(E) = [t_1, t_n]$ and $rangeInterval(E) = [v_{\min}, v_{\max}]$.

### 3.1 Measures of Variation

We will work with pairs of successive values $(v_1, v_2), (v_2, v_3), ...(v_i, v_{i+1})...$ On this basis, we define a *measure of variation* that will turn out to be the critical measure in measuring change and continuity. The measure is defined as the sum of the absolute values of successive pairs in the event set:

$$Sum(E) = \sum_i \; | v_{i+1} - v_i |$$

A second measure is defined as the maximum of the absolute values of successive pairs in the event set:

$$Max(E) = \max_i | v_{i+1} - v_i |$$

The sum of variations turns out to be activity measure as previously defined in the context of differential equation simulation [3]. The max measure allows us to characterize the uncertainty in a data stream and the smallest quantum that can be used in quantization of it.

### 3.2 Extrema – Form Factor of an Event Set

The *form factor* of an event set consists of the values and locations of its extrema. This information is represented in an event set as follows:

$extrema(E) = \{(t*_i, v*_i)\} \subseteq E$

where $(t* \;, v* \;)$ represents a maximum or minimum at location $t*$ with value $v*$.

A subsequence $(t_i, v_i)(t_{i+1}, v_{i+1}), ..(t_{i+m}, v_{i+m})$ is monotonically increasing if $v_i > v_{i+1} > .. > v_{i+m}$. The sequence is non-decreasing if $v_i \geq v_{i+1} \geq .. \geq v_{i+m}$.

A similar definition holds for the terms monotonically decreasing and non-increasing. An algorithm to obtain the form factor proceeds from the following:

**Theorem**: 1 Let $E$ be an event set. Then its minima and maxima alternate, one following the other. The subsequences between successive extrema are either non-increasing or non-decreasing, according to whether they start with a minimum or a maximum.

Corollary: 1 An event set can be decomposed into a "disjoint" union of monosets. We use the quotation marks "disjoint" to indicate disjointness except for overlapping end points.

Proposition: 1 The sum of variations in a monoset $E$ can be expressed as:

$$Sum(E) = | v_1 - v_n |$$

**Theorem**: 2
For an event set $E$ with $extrema(E) = \{(t*_i, v*_i)\}$, the sum of variations, $Sum(E) = \sum_i | v*_{i+1} - v*_i |$.
In other words, $Sum(E)$ is the sum of its monoset sums of variation.

### 3.3 Refinement

Refinement is a process by which we add new data to an existing event set without modifying its existing event pairs. The result of refinement is a new event set that is said to refine the original one. We define the relation

$E$ *refines* $E'$ iff $E \supseteq E'$, i.e., the set of event pairs in $E$ includes the set of events in $E'$.

Proposition: 2 $E$ *refines* $E' \Rightarrow Sum(E) \geq Sum(E')$

**Proof**: Since $E$ *refines* $E'$ there is a pair, $(t', v')$ squeezed between some pair $(t_i, v_i), (t_{i+1}, v_{i+1})$ ,i.e., $t_i < t' < t_{i+1}$ .Then $| v_{i+1} - v_i | \leq | v_{i+1} - v' | + | v' - v_i |$.

### 3.4 Within-the-box Refinement

We now identify precisely the type of refinement that does not increase the sum of variations. A refinement is *within-the-box* if the added pair $(t', v')$ satisfies the constraints: $t_i < t' < t_{i+1}$ and $v' \in [\min(v_i, v_{i+1}), \max(v_i, v_{i+1})]$. Then we have:

$E$ *refines* wtb $E'$ if $E$ *refines* $E'$ and all refinement pairs are within-the-box.

Within-the-box refinement has a special property – it preserves the sum and tends to decrease the maximum variation. As we shall see, this simple rule characterizes refinement of continuous functions once their form factors have been identified,.

Proposition: 3 Assume that $E$ *refines* $E'$. Then $Sum(E) = Sum(E')$ if, and only if, $E$ *refines* wtb $E'$,
Also $E$ *refines* wtb $E' \quad \Rightarrow Max(E) \le Max(E')$, but the converse is not true.

**Theorem: 3** The monoset decomposition of an event set is not altered by within-the-box refinement. Consequently, an event set's form factor is invariant with respect to within-the-box refinement.

**Proof**: A within-the-box single refinement falls into some monoset domain interval. It is easy to see that the within-the-box refinement does not change the non-increasing (or non-decreasing) nature of the monoset sequence. In particular, it does change the values and locations of the extrema.

## 4 Domain and Range-Based Event Sets

An event set $E = \{(t_i, v_i) \mid i = 1...n\}$ is domain-based if there is a fixed step $\Delta t$ such that $t_{i+1} = t_i + \Delta t$ for $i = 1...n-1$ We write $E_{\Delta t_*}$ to denote a domain-based event set with equally spaced domain points separated by step $\Delta t$ .

An event set $E = \{(t_i, v_i) \mid i = 1...n\}$ is *range-based* if there is a fixed quantum $q$ such that $| v_{i+1} - v_i | = q$ for $i = 1...n-1$. We write $E_q$ to denote a range-based event set with equally spaced range values, separated by a quantum $q$ .

For a range-based event set $E_q$, the measures of variation are simply expressed as $Sum(E_q) = size(E_q)q$ and $Max(E_q) = q$ . For a domain-based event set $E_{\Delta t}$, we have

$$\Delta t = \frac{domainIntervalLength}{size(E_{\Delta t})}$$

We can map between the two forms of representation. If we are interested in what happens in a behavior at specific times than the domain-based representation is more appropriate. However, usually we are interested more in recording only times of significant changes and inferring that nothing of interest occurs between such events. In this case, the range-based representation is appropriate and has significant economy of representation. We next see the application of this principal to continuous function representation.

## 5 Event Set Representation of Continuous Functions

Let $f:[t_i,t_f]\to R$ be a continuous function defined everywhere on the closed interval $[t_i,t_f]$ with a finite number of alternating minima and maxima separated by non-increasing, and non-decreasing segments. Surprisingly, perhaps, it can be shown that differentiable continuous functions, except for those with damped infinite oscillations, are of this form. Let $extrema(f)$ denote the extrema of $f$.

Definition: $E(f)$ is an event set that is a sample of $f$ if $(t,v)\in E(f)\Rightarrow f(t)=v.$. We say $E(f)$ samples $f$. In addition, if $E(f)\supseteq extrema(f)$ we say $E(f)$ *represents* $f$.
The smallest event set able to represent $f$ is $E_{extrema}(f)=extrema(f)$. Almost by definition, if any $E(f)$ *represents* $f$ then it refines $E_{extrema}(f)$.

If $E'(f)$ refines $E(f)$ and samples $f$ we say that $E'(f)$ is a *continuation of* $E(f)$. The relation "is a continuation of" is transitive with minimal element $E_{extrema}(f)$ ,i.e., every event set that represents $f$ is a continuation of $E_{extrema}(f)$.

Our fundamental connection to within-the-box refinement is given in:

**Theorem: 4 Let** $E(f)$ represent $f$.Then

1. Every extremum of $f$ is an extremum in $E(f)$, and conversely.
2. $Sum(E(f))=Sum(f)=\sum_i |v^f{}_{i+1}-v^f{}_i|$
3. Every continuation of $E(f)$ is a within-the-box refinement.
4. For every $E'(f)$ continuation of $E(f)$, we have:
   a. $Sum(E'(f))=Sum(E(f))$
   b. $Max(E'(f))\le Max(E(f))$.

### 5.1 Uncertainty Metric

We introduce a metric to facilitate a comparison of the number of samples required to attain a given accuracy of representation. Since samples are assumed to be noise-free, the remaining imprecision is in the uncertainty that a finite set of samples implies about the rest of the points in the function. For a given domain point, we take its uncertainty as the size of the interval in which its mapped value is known to lie.

Definition: The *box* in $E(f)$ containing $(v,t) \in f$ is spanned by the domain sub-interval $(t_i, t_{i+1})$ containing $t$ and the associated range sub-interval $(v_i, v_{i+1})$. Define $\text{uncertainty}(E(f),t) = |v_i - v_{i+1}|$ where $(v_i, v_{i+1})$ is vertical side of the box in $E(f)$ containing $(v,t)$. Now

$$\text{uncertainty}(E(f)) = \max\{\text{uncertainty}(E(f),t) \mid t \in domainInterval(E(f)\}$$

Proposition: 4

1. $\text{uncertainty}(E(f)) = Max(E(f))$.
2. For a range-based continuation with quantum $q$, $\text{uncertainty}(E(f)) = q$
3. If $f$ is differentiable, then for a domain-based continuation with small enough step $\Delta t$, we have

   $$\text{uncertainty}(E(f)) \approx MaxDer(f) * \Delta t$$

where $MaxDer(f)$ is the magnitude of the largest derivative of $f$.

**Proof**: Assertions 1 and 2 follow easily from the definitions. For Assertion 3, we note that for successive values, $v_i, v_{i+1}$ with small enough domain step $t_{i+1} - t_i$ we have $|v_i - v_{i+1}| \approx |f'(t_i)|(t_{i+1} - t_i)$, where $f'(t)$ is the derivative we have assumed to exist. The rest follows easily.

**Theorem: 5** The sizes of domain-based and range-based continuation representations of a differentiable function $f$ having the same uncertainty are related as follows:

$$R = \frac{size(E_q(f))}{size(E_{\Delta t}(f))} \approx \frac{Avg(f)}{MaxDer(f)} \leq 1$$

where $Avg(E(f)) = \dfrac{Sum(f)}{domainIntervalLength} = Avg(f)$.

We see that the number of points required for a range-based representation is proportional to its sum of variations. This is the general statement of the result proved in previous work for the activity in differential equation systems.

### 5.2 The. Range-Based Representation Can Be Arbitrarily More Efficient Than Its Domain-Based Equivalent

**Theorem: 6** There are smooth continuous functions whose parameters can be set so that any range-based representation uses an arbitrarily smaller number of samples than its domain-based equivalent (by equivalent, we mean that they achieve the same uncertainty.)

In Table 1 we compare sizes for representations having the same uncertainty. Except for the sine wave, there are parameters in each of the classes of functions that can be set to achieve any reduction ratio for range-based versus domain-based representation. In the case of simulation, this implies that the number of computations is much smaller hence the execution goes much faster for the same uncertainty or precision. Indeed, we can extend the event set concept to multiple dimensions with the appropriate definitions, and obtain:

**Theorem: 7** The sizes of domain-based and range-based continuation representations of a differentiable n-dimensional function $f = \langle f_1, \ldots, f_n \rangle$ having the same uncertainty are related as follows: $R = \frac{size(E_q(f))}{size(E_{\Delta t}(f))} \approx \frac{Avg(f)}{n * MaxDer(f)} \le 1$.

Calculations similar to those in Table 1 show that a performance gain that is proportional to the number of dimensions is possible when there is a marked in- homogeneity of the activity distributions among the components.

**Table 3.** Compuation of Range-based to Domain-based Representation Size Ratios

| $f(t) =$ | $domainInterval$ | $Sum(f)$ | $Avg(f) = \frac{Sum(f)}{domain\ IntervalLength}$ | $MaxDer(f)$ | $R = \frac{size(E_q(f))}{size(E_{\Delta t}(f))} \approx \frac{Avg(f)}{MaxDer(f)}$ |
|---|---|---|---|---|---|
| $t^n, n \ge 1$ | $[0,T]$ | $T^n$ | $T^{n-1}$ | $nT^{n-1}$ | $1/n$ |
| $t^{-n}, n \ge 1$ | $[1,T]$ | $1-T^{-}$ | $\frac{1-T^{-n}}{T-1}$ | $nt^{-(n+1)} \mid 1 = n$ | $\frac{1-T^{-n}}{n(T-1)}$ $\lim_{T\to\infty} = \frac{1}{n}$ |
| $A \sin \omega t$ | $[0, \frac{2\pi}{\omega}]$ | $4A$ | $\frac{2A\omega}{\pi}$ | $A\omega$ | $\frac{2}{\pi}$ |
| $Ae^{-at}$ | $[0,T]$ | $A(1-e^{-}$ | $A(1-e^{-aT})/T$ | $Aa$ | $\frac{1-e^{-aT}}{aT}$ $\lim_{T\to\infty} = \frac{1}{aT}$ |

For example, for a signal $f_i(t) = a_i \sin(\omega_i t), i = 1..n$, the ratio R= $\frac{2\sum_{i=1}^{n} \omega_i a_i}{n\pi \max_i \{\omega_i a_i\}}$ varies as $\frac{2}{\pi} \le R \le \frac{2}{n\pi}$, For an n-th degree polynomial we have $\frac{1}{n^2} \le R \le \frac{1}{n}$. So that potential gains of the order of $O(n^2)$ are possible.

## 6 Event Set Differential Equations

Although space precludes the development here, the integral and derivative operations can be defined on event sets that parallel those in the traditional calculus. Assuming this development, we can formulate the event set analog to traditional differential equations as follows:

Let $E_i = \{(t_j, v_j) \mid j = 1...n\}$ be required to be an indefinite sequence of within-the-box refinements that satisfies the equation: $D\ (E_i) = f(E_i)$ for all $i = 1, 2, ...$ where

- $D\ (E)$ is the derivative of $E$ and
- $f(E) = \{(t_i, f(v_i)) \mid i = 1...n\}$ with $f : rangeInterval(E) \rightarrow R$

A solution to this equation is

$$E_q^{solution} = \{(t_{q,j}, jq) \mid j = 1...rangeInterval / q\}$$

where $t_{q,j} = I(G)_{q,j}$ and $I(G)$ is the integral of $g(v) = 1/f(v)$, i.e., $I(G)_{q,j} = \{(jq, I(G_{q,j})\}$. We note that the solution is a range-based event-set that can be generated by DEVS quantized simulation. It parallels the solution to $\frac{dx}{dt} = f(x)$ written as $\int_{x(t_i)}^{x(t)} \frac{dx}{f(x)} = t - t_i$. It turns out that the range-based event set concept is essential to writing explicit solutions that parallel the analytic solutions of classical calculus. Recall that without going to the state description $\frac{dx}{f(x)} = dt$ ,we can only state the recursive solution $F(t) = \int^t f(F(\tau))d\tau$. Likewise, we can't write an explicit solution for the event set differential equation without going to the range-based, i.e., quantized formulation.

## 7 Discussion and Applications

Originally developed for application to simulation of ordinary, and later, partial differential equations, the activity concept has herein been given a more general and fundamental formulation. Motivated by a desire to reconcile everyday discrete digital computation with the higher order continuum of traditional calculus, it succeeds in reducing the basics of the latter to computer science without need of analytical mathematics. A major application therefore is to the revamping education in the calculus to dispense with its mysterious tenets that are too difficult to convey to learners. An object-oriented implementation is available for beginning such a journey.

Other applications and directions are:

- **sensing**– most sensors are currently driven at high sampling rates to obviate missing critical events. Quantization-based approaches require less energy and produce less irrelevant data.
- **data compression** – even though data might be produced by fixed interval sampling, it can be quantized and communicated with less bandwidth by employing domain-based to range-based mapping.
- **reduced communication** in multi-stage computations, e.g., in digital filters and fuzzy logic is possible using quantized inter-stage coupling.
- **spatial continuity**–quantization of state variables saves computation and our theory provides a test for the smallest quantum size needed in the time domain; a similar approach can be taken in space to determine the smallest cell size needed, namely, when further resolution does not materially affect the observed spatial form factor.
- **coherence detection** in organizations – formations of large numbers of entities such as robotic collectives, ants, etc. can be judged for coherence and maintenance of coherence over time using this paper's variation measures.

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